# Raman Spectra and Valence Force Field of Single-Crystalline $\boldsymbol{\beta} \mathbf{G a}_{2} \mathbf{O}_{3}$ 

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#### Abstract

Single-crystalline $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ was studied by Raman spectroscopy between 10 and 1000 K . A complete valence force field was obtained. The agreement between observed and calculated Raman active frequencies is $0.8 \%$ for the whole spectrum. The potential energy distribution shows that stretching and bending modes are not independent. A complete description of the modes is given in terms of cartesian displacements. The temperature dependence of the Raman band half-widths was studied and is discussed in terms of anharmonicity at high temperature; the broadening which appears above 100 K seems to be related to the appearance of defects. Our spectra are better understood in terms of the $C 2 / m$ space group proposed by Geller rather than $P 1$ as recently proposed by G. M. Wolten and A. B. Chase (J. Solid State Chem. 16, 377, 1976).


## I. Introduction

This work was undertaken as part of a general study of oxides, in particular those belonging to the $\beta$ alumina family. Like that of $\beta$ aluminates ( $11 \mathrm{Al}_{2} \mathrm{O}_{3}-\mathrm{M}_{2} \mathrm{O}$ ) and $\beta$ gallates, the structure of $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ is made of $\mathrm{GaO}_{6}$ (edge sharing) octahedra and $\mathrm{GaO}_{4}$ (corner sharing) tetrahedra. Preliminary Raman results ( $1 a$ ) have shown that the $\beta$ $\mathrm{Ga}_{2} \mathrm{O}_{3}$ spectra are much simpler than those of stoichiometric $\beta$ gallates, due to the fact that the unit cell multiplicity is much lower in the former. In these conditions it seemed interesting to undertake a complete infrared and Raman study of a single crystal of $\beta$ $\mathrm{Ga}_{2} \mathrm{O}_{3}$ in order to deduce a reliable valence
force field transferable to $\beta \mathrm{Al}_{2} \mathrm{O}_{3}$ allowing the interpretation of the spectra of this compound ( $1 b, 2$ ). The structure of $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ reported by Geller (3) has been criticized recently by Wolten and Chase (4). Therefore it seemed of interest to look for structural information from spectroscopy experiments. There are only a few spectroscopic data (5) which are of no use for the assignment of the different bands either in terms of symmetry or in terms of internal coordinates.

Moreover, although no first-order phase transition has been detected by DTA in the 200 to $500-\mathrm{K}$ temperature range ( $6 a$ ) it seemed interesting to investigate the role played by the anharmonicity and also the
existence of possible phase transitions in the $10-$ to $1000-\mathrm{K}$ range. These phenomena are usually detected from the temperature dependence of frequencies and half-widths of Raman and ir bands.

Finally, this work was undertaken as the first step of a study of a series of $\mathrm{Ga}_{2} \mathrm{O}_{3}$ crystals doped with ions such as $\mathrm{Cr}^{3+}$, having possible lasing properties. Preliminary Raman results ( $6 b$ ) corroborate ESR results (3), i.e., for concentrations as high as $3 \% \mathrm{Cr}^{3+}$, the crystal structure remains ordered, indicating that $\mathrm{Cr}^{3+}$ substitutes $\mathrm{Ga}^{3+}$ ions.

## II. Experimental

1. Crystal growth. $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ was prepared by heating $99.9999 \%$ gallium hydroxide $\mathrm{GaO}(\mathrm{OH})$ in air at $1000^{\circ} \mathrm{C}$ for 12 hr . Single crystals were grown by a floating zone method ( $6 c$ ), associated with an image furnace in which the light source consists of a $6.5-\mathrm{kW}$ xenon lamp.

A crystal was cut and oriented following indicatrix axes. A parallepiped of $4 \times 2 \times 1$ mm was obtained for which the $C_{2}$ axis ( $b$ axis) was taken as the $Z$ axis (Fig. 1). The axis direction was verified through Laüe


Fig. 1. Relation between crystallographic $a, b, c$ axes and observed $X, Y, Z$ axes for the $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ single crystal $(4 \times 2 \times 1 \mathrm{~mm})$ used in this study.
patterns (a polarizing microscope observation was not sufficient). The $Y$ axis coincides with the $a$ crystallographic direction: it is parallel to the $n_{m}$ axis of the indices ellipsoid. The $X$ axis was taken orthogonal to the $Y$ and $Z$ axes; it defines an angle of $13.7^{\circ}$ with the $c$ axis in the (ac) plane.
2. Spectra. Polarized Raman spectra of the single-crystalline sample were recorded on a Coderg T 800 instrument equipped with an ionized argon laser. The polarizations reported in Fig. 4 and Table I are based on the Porto's notation. Leaks into unallowed polarization are less than $2 \%$. Low-temperature spectra ( $10-300 \mathrm{~K}$ ) were obtained by immersing the crystal in $4-\mathrm{K}$ gaseous helium, the temperature being controlled by thermocouple. At high temperature $(300-1000 \mathrm{~K})$ four independent series of measurements were performed in order to obtain precise curves representing the half-width variations as a function of temperature. Measurements were carried out every 25 K between 4 and 673 K for the most sensitive modes. The curve corresponding to the $353-\mathrm{cm}^{-1}$ mode is not reported, because the $345-\mathrm{cm}^{-1}$ mode overlaps this band and the deconvolution is not reliable for large half-widths.

## III. Structure and Selection Rules

According to Geller, $\beta \quad \mathrm{Ga}_{2} \mathrm{O}_{3}$ has a monoclinic structure and belongs to the $C_{2 h}^{3}$ space group; the unit cell contains two formula units (Fig. 2b). There are two crystallographically nonequivalent gallium and three nonequivalent oxygen ions in the unit cell. Each $\mathrm{Ga}_{\mathrm{i}}{ }^{3+}$ ion is surrounded by a distorted tetrahedron of oxygen ions: one $\mathrm{O}_{1}^{2-}$ ion at $1.80 \AA$, two $\mathrm{O}_{\mathrm{II}}^{2}$ ions at $1.83 \AA$, and one $\mathrm{O}_{\text {III }}^{2}$ at $1.85 \AA$. Each $\mathrm{Ga}_{\text {II }}^{3 \text { i }}$ ion is surrounded by a distorted octahedron of oxygen: two $\mathrm{O}_{1}^{2-}$ at $1.95 \AA$, one $\mathrm{O}_{\mathrm{II}}^{2}$ at $1.95 \AA$, two $\mathrm{O}_{\text {III }}^{2}$ at $2.08 \AA$, and one $\mathrm{O}_{\text {III }}^{2}$ at $2.02 \AA$. A tetrahedron shares only corners with other tetrahedra in the $b$ axis direction and with

TABLE I
Infrared and Raman Spectra of $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ at Different Temperatures

| Infrared ${ }^{a}$ <br> (K) |  | Raman (K) |  |  | Polarization relative intensities ${ }^{b}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 300 | 10 | 300 | 800 | XX | YY | ZZ | XY | YZ | XZ |
|  |  |  | 111 | 108 | 45 | - | * | - | - | * |
|  |  | 115 | 114 | 112 | - | - | - | - | 10 | 20 |
|  |  | 147 | 147 | 140 | * | - | - | * | 70 | 45 |
|  | 155 m | 170 | 169 | 165 | 135 | 65 | 55 | - | * |  |
|  | 155 m | 200 | 199 | 199 | 1000 | 175 | 315 | 15 | 5 | 15 |
|  | 250 m |  |  |  |  |  |  |  |  |  |
|  | 275 m |  |  |  |  |  |  |  |  |  |
|  | 290 m |  |  |  |  |  |  |  |  |  |
|  | 310 sh |  |  |  |  |  |  |  |  |  |
|  |  | 319 | 318 | 317 | 45 | 20 | 20 | 25 | * | * |
|  |  | 347 | 346 | 340 | 20 | 185 | 120 | * | * | * |
|  |  | 355 | 353 | 340 | - | - | - | - | 15 | * |
| 375 S | 375 m |  |  |  |  |  |  |  |  |  |
|  |  | 415 | 415 | 414 | 125 | 175 | 110 | 35 | * | * |
| 455 S | 455 S |  |  |  |  |  |  |  |  |  |
|  |  | 475 | 475 | 468 | 75 | * | 60 | * | * | 40 |
| 525 sh | 525 sh |  |  |  |  |  |  |  |  |  |
|  |  | 629 | 628 | 620 | 115 | * | 25 | 10 | * | * |
| 640 sh | 640 sh |  |  |  |  |  |  |  |  |  |
|  |  | 651 | 651 | 640 | - | - | - | - | 10 | 110 |
|  |  | 657 | 657 | 648 | * | 125 | * | 70 | - | - |
| 668 S | 668 F |  |  |  |  |  |  |  |  |  |
| 720 sh | 720 sh |  |  |  |  |  |  |  |  |  |
| 760 m |  |  |  |  |  |  |  |  |  |  |
|  |  | 767 | 763 | 754 | 65 | 595 | - | 25 | * | * |

${ }^{a} S$ : strong; m: mean, sh: shoulder.
${ }^{b}$ Raman spectra of a single crystal at 300 K : relative intensities for different polarizations. XX, YY, ZZ, XY are the elements of scattering matrices corresponding to $A_{g}$ modes, YZ and XZ are relative to $B_{g}$ modes; an intensity of 1000 has been given to the XX component of the $199-\mathrm{cm}^{-1}$ band.
${ }^{*}$ Indicates bands of low intensity probably due to polarization leaks into unallowed components ( $<2 \%$ ).


Fig. 2. (a) Representation of the tetrahedra and octahedra which form the structure of $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$. (b) Schematic representation of the unit cell of $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$; the atoms which are considered to define the $k=0$ motions are included inside the broken line.
octahedra in other directions (Fig. 2a). An octahedron shares edges with adjacent octahedra. Each $\mathrm{O}_{1}^{2-}$ ion is at the corner of two octahedra and one tetrahedron; each $\mathrm{O}_{\text {II }}^{2}$ is at the corner of one octahedron and two tetrahedra. Each $\mathrm{O}_{\text {III }}^{2}$ ion is at the corner of three octahedra and one tetrahedron.

This structure has been criticized by Wolten and Chase. These authors claim that $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ crystals are twinned and that they belong to the triclinic system and that their structure is described by the noncentrosymmetric space group $C_{1}^{1}$ with two formula units per cell.

Under the $C_{2 h}$ factor group the $k=0$ crystal modes can be classified according to

$$
\Gamma=10 A_{g}+5 B_{g}+10 B_{u}+5 A_{u}
$$

from which acoustic modes $\left(A_{u}+2 B_{u}\right)$ must be substracted. Therefore 15 Raman and 12 infrared active modes are expected. Under the $C_{1}$ factor group one expects 27 modes both infrared and Raman active.

When only the shortest $\mathrm{Ga}-\mathrm{O}$ bonds are considered, one can define the structure as made of chains of $\mathrm{GaO}_{4}$ tetrahedra between which one finds "isolated" $\mathrm{Ga}_{\text {II }}$ ions. Furthermore, one can discriminate between $\mathrm{Ga}_{\mathrm{I}}\left(\mathrm{O}_{\mathrm{I}}\right)_{2}$ groups of atoms located in (010) planes and $\mathrm{Ga}_{\mathrm{I}}\left(\mathrm{O}_{\mathrm{II}}\right)_{2}$ groups constituting chain links (Fig. 2b).

This approximate description is useful to describe the normal modes of the crystal; moreover, it is consistent with the fact that force constants decrease when the bond lengths increase, i.e., long bonds do not contribute at a large extent to the vibrational energy. For each $\mathrm{Ga}_{1} \mathrm{O}_{4}$ group of $C_{8}$ site symmetry, one expects three stretching modes: on the one hand there are $\nu_{s} \mathrm{GaO}_{2}$ of $A^{\prime}$ symmetry and $\nu_{a} \mathrm{GaO}_{2}$ of $A^{\prime}$ symmetry which describe the in-plane symmetric and antisymmetric stretching modes of the $\mathrm{Ga}_{1}\left(\mathrm{O}_{\mathrm{I}}\right)_{2}$ groups, respectively; on the other hand there is the $\nu \mathrm{Ga}_{1}\left(\mathrm{O}_{\mathrm{II}}\right)_{2}$ mode describing the stretching of the bonds belonging to the chains. For $k=0$ only one stretching
mode remains because of the fact that the two oxygen atoms connected to a given $\mathrm{Ga}_{\mathrm{I}}$ ion are congruent. In the same way bending modes can be defined for the tetrahedra. These are reported in Table II and Fig. 3 gives a description of these above-discussed vibrations. Motions of oxygen atoms common to tetrahedra and octahedra are considered deliberately as belonging to tetrahedra vibrations, therefore the pure octahedra vibrations are generated by the $\mathrm{Ga}_{\text {II }}$ translations. Thus 22 internal modes are described. Finally, by considering the $R_{y}^{\prime}$ librations of chains around the $b$ axis and the three translations of each chain, one obtains eight external modes.

## IV. Results and Discussion

## 1. Qualitative Assignments of Spectra

By considering Fig. 4 one can find ten bands in the XX polarization and five bands for the $X Z$ spectrum. These bands appear at different frequencies with the exception of the $475-\mathrm{cm}^{-1}$ band which is observed in

TABLE II
Symmetry Correlation for Localized Coordinates

| Number | Localized coordinates |  | Spacc group $C_{\text {2h }}$ |
| :---: | :---: | :---: | :---: |
| 1 | $\nu_{s} \mathrm{Ga}_{1}\left(\mathrm{O}_{1}\right)_{2}$ | $A^{\prime}$ | $A_{g}+B_{k}$ |
| 2 | $\delta \mathrm{Ga}_{1}\left(\mathrm{O}_{\mathrm{II}}\right)_{2}$ | $A^{\prime}$ | $A_{g}+B_{k}$ |
| 3 | $\nu_{a} \mathrm{Ga}_{1}\left(\mathrm{O}_{\mathrm{II}}\right)_{2}$ | $A^{\prime \prime}$ | $A_{v}+B_{\square}$ |
| 4 | $\nu_{a} \mathrm{Ga}_{1}\left(\mathrm{O}_{1}\right)_{2}$ | $A^{\prime}$ | $A_{g}+B_{v}$ |
| 5 | $\delta \mathrm{Ga}_{1}\left(\mathrm{O}_{\mathrm{I}}\right)_{2}$ | $A^{\prime}$ | $A_{o}+B_{u}$ |
| 6 | Rock. ${ }_{1} \mathrm{Ga}_{\mathbf{4}}\left(\mathrm{O}_{1}\right)_{2}$ | $A^{\prime \prime}$ | $A_{*}+B_{q}$ |
| 7 | Rock. $\\| \mathrm{Ga}_{4}\left(\mathrm{O}_{1}\right)_{2}$ | $A^{\prime}$ | $A_{g}+B_{u}$ |
| 8 | Twist. $\mathrm{Ga}_{4}\left(\mathrm{O}_{1}\right)_{2}$ | $A^{\prime \prime}$ | $A_{k}+B_{0}$ |
| 9 | Translations of $\int T_{x}$ | $A^{\prime}$ | $A_{0}+B_{u}$ |
| 10 | $\mathrm{Ga}_{\text {II }}\left\{\begin{array}{l}T_{u} \\ T_{z}\end{array}\right.$ | $A^{\prime \prime}$ | $A_{x}+B_{0}$ |
| 11 | (octahedron modes) $T_{z}$ | $A^{\prime}$ | $A_{\nu}+B_{*}$ |
| 12 | Libration of chains $R_{y}^{\prime}$ | $A^{\prime}$ | $A_{y}+B_{x}$ |
| 13 | Translations $\quad \begin{aligned} & T_{r}^{\prime} \\ & T_{r}^{\prime}\end{aligned}$ | $A^{\prime}$ | $A_{s}+B_{s}{ }^{\text {a }}$ |
| 14 | of $\quad\left\{\begin{array}{l}T_{y}^{\prime} \\ T_{z}^{\prime}\end{array}\right.$ | $A^{\prime \prime}$ | $A_{\text {c }}{ }^{\text {a }}+B_{\text {a }}$ |
| 15 | chains $\quad T_{z}^{\prime}$ | $A^{\prime}$ | $A_{0}+B_{u^{\text {a }}}{ }^{\text {a }}$ |

[^0]

FIg. 3. Localized vibrations (chemically significant).
both ( $A_{g}$ and $B_{g}$ ) symmetry classes. It is interesting to note that polarization measurements allow the separation of the 111to $114-$ and $345-$ to $353-\mathrm{cm}^{-1}$ components. In other words, the $10 A_{g}+5 B_{g}$ modes expected for the $C_{2 n}$ factor group are observed. This is confirmed by the low-temperature Raman spectrum (the crystal was misoriented on purpose) which contains 11 strong bands and two shoulders at about 350 and $660 \mathrm{~cm}^{-1}$, respectively. Moreover, infrared and Raman exclusion can be checked on Fig. 5. It is possible to count 11 infrared maxima (the very weak bond at 275 $\mathrm{cm}^{-1}$ is probably due to an overtone) assignable to the 12 expected infrared active modes. These observations agree with the $C_{2 h}$ centrosymmetric structure proposed by Geller (3) rather than with the $C_{1}$ factor group proposed recently by Wolten and Chase (4). Table I reports the relative intensities for the different polarizations for each Raman band along with the infrared frequencies. Their assignment in terms of symmetry is reported unambiguously for Raman active modes.

A qualitative assignment can be proposed in order to define spectral regions in which it is reasonable to expect characteristic motions of the crystal before performing any calculations. In this respect three regions can be identified in the Raman spectra. The first region below $200 \mathrm{~cm}^{-1}$ is well separated from the high-frequency region by a gap of $100 \mathrm{~cm}^{-1}$. This region is characterized by narrow bands ( $\simeq 3 \mathrm{~cm}^{-1}$ at 300 K ) as compared to the other modes $\left(7 \mathrm{~cm}^{-1}\right)$. These


Fig. 4. Polarized Raman spectra of a single crystal of $\boldsymbol{\beta} \mathrm{Ga}_{2} \mathrm{O}_{3}$ at 300 K .


Fig. 5. Infrared and Raman spectra of polycrystalline $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ at low temperature.
bands must be related to lattice modes corresponding to motions of small amplitude, such as vibrations involving the tetrahedra chains as a whole (translation and libration motions). The frequencies above $200 \mathrm{~cm}^{-1}$ should belong to vibrations which can be considered as internal vibrations, i.e., bending and stretching modes of tetrahedra groups. Considering the chemical description reported in the structural part of this article and the approximation of localized vibration coordinates (Fig. 3 and Table II), one can expect that stretching modes of tetrahedra occur at high frequency; in particular it seems reasonable to assign the highest frequency observed at $763 \mathrm{~cm}^{-1}$ to a stretching mode involving $\mathrm{Ga}-\mathrm{O}_{\mathrm{I}}$ bonds, which are the shortest ones. The bending modes of tetrahedra are expected in the range $400-600 \mathrm{~cm}^{-1}$, while vibrations involving motions of octahedra (translations of $\mathrm{Ga}_{\mathrm{II}}$ ) take place probably below 400 $\mathrm{cm}^{-1}$.

## 2. Valence Force Field Calculations

Calculations were performed with a program of Shimanouchi's type (7). The force constants matrix was set up by using
weighted symmetrical cartesian coordinates.

In the calculation 50 atoms were taken into account: those of the unit cell and their closest neighbors. Cartesian coordinates were deduced from the X-ray data reported by Geller (3). Two oxygen atom families and two types of gallium atoms were considered to define internal coordinates. The 11 internal coordinates are divided into four GaO stretching coordinates $\left(f_{\mathrm{t1}}, f_{\mathrm{t} 2}, f_{\mathrm{o} 1}\right.$, $f_{02}$ ), six bending coordinates ( $f \delta_{1}, f \delta_{2}, f \theta_{1}$, $f \theta_{2}, f \theta_{3}, f \gamma$ ), and a torsional coordinate $\tau$ (around the Ga-O bonds). These coordinates are represented in Fig. 6.

We considered 29 distinct force constants (11 diagonal and 18 interaction force constants). For every normal mode, cartesian displacements (LX) and potential energy distribution (PED) were calculated. Several force constants sets were tested; in particular, we tried a linear dependence of the four valence force constants as a function of the $\mathrm{Ga}-\mathrm{O}$ distance. A refinement using leastsquares method was performed; however, the correlation matrices have shown that force constants related to internal coordinates of octahedra were strongly dependent on those characteristic of tetrahedra (for example, force constants $1-5$ were correlated with a factor of -0.62 ). Finally, the best agreement between experiments and calculations was obtained by letting valence force constants vary freely. In these conditions we observed a linear relationship $\nu=$ $f(d)$ for two stretching constants of octahedra and one of tetrahedra. The second tetrahedra force constant is lower than what could be expected from this relationship (Fig. 7). Table III gives the final values for force constants. The atoms involved in the associated internal coordinates are also reported. A remarkable agreement between calculated and observed Raman frequencies is obtained. The error on the whole spectrum is less than $0.8 \%$. On the other


Fig. 6. Definition of internal coordinates used in the valence force field calculation. The primitive cell is inside the broken line.
hand, the agreement between infrared frequencies and $A_{u}, B_{u}$ calculated frequencies is not so good, except for the narrow bands measured at 155 and $372 \mathrm{~cm}^{-1}$. This disagreement is not surprising when one realizes that the infrared spectrum was recorded on a polycrystalline sample dispersed in nujol. The infrared bands corresponding to strongly polar modes lie between TO and LO frequencies (in between is the Reststrahlen region); therefore the observed maxima cannot coincide with the calculated frequencies. We can notice that the transverse mode for which the macro-


Fig. 7. Relationship between the stretching force constants and the bond length for $\mathrm{Ga}-\mathrm{O}$ bonds: $f\left(\right.$ mdyne $\left.\AA^{-1}\right)=-8.628 d(\AA)+18.05$.
scopic electric field is weak must lie nearby the calculated frequency.

Moreover, we confirm that the Raman band at $475 \mathrm{~cm}^{-1}$ is a superposition of two modes of $A_{g}$ and $B_{g}$ symmetry, the corresponding calculated frequencies being 474 and $468 \mathrm{~cm}^{-1}$, respectively.

Two types of stretching force constants are found. The first one concerns the $\mathrm{Ga}-\mathrm{O}$ bonds located in the ( 010 ) mirror plane and those of the (001) plane containing the axes of tetrahedra chains. They represent the strong bonds of the crystal. The second type corresponds to $\mathrm{Ga}-\mathrm{O}$ bonds belonging to the square plane of the octahedron (defined by atom numbers $11,12,13$, and 14 in Fig. 6) and is weaker ( 0.6 mdyne $\AA^{-1}$ ) than the preceding one. The (100) and (001) planes are two cleavage planes in $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ ( $6 c$ ). The results obtained from this valence force calculation show that the most probable cleavage planes are those which do not intersect chains axes, i.e., the (100) and (001) planes. Moreover, the ( 001 ) plane seems to be the easiest plane for cleavage. This can be seen from the following considerations. The potential associated with a definite bond can be described by a Morse function $V_{\mathrm{n}}=D_{\mathrm{e}}\left(1-e^{-\beta x}\right)^{2}$. The force

TABLE III
Definition of Force Constants

Number \begin{tabular}{ccc}
\hline Force <br>

constants \& \begin{tabular}{c}
Calculated <br>
values <br>
$(m d y n e ~$ <br>
$\left.\AA^{-1}\right)$

 \& 

Examples of atoms <br>
involved in the <br>
corresponding internal <br>
coordinate
\end{tabular} <br>

\hline
\end{tabular}

| Stretching |  |  |  |
| :---: | :---: | :---: | :---: |
| 1 | $f_{4}$ | 2.25 | $\mathrm{Ga}_{1}-\mathrm{O}_{3}$ |
| 2 | $f_{a_{1}}$ | 0.60 | $\mathrm{Ga}_{4}-\mathrm{O}_{13}$ |
| 3 | $f_{l_{2}}$ | 1.62 | $\mathrm{Ga}_{1}-\mathrm{O}_{7}$ |
| 4 | $f_{\mathrm{O}_{2}}$ | 1.27 | $\mathrm{Ga}_{3}-\mathrm{O}_{9}$ |
| Bending |  |  |  |
| 5 | $f \theta_{1}$ | 0.45 | $\mathrm{O}_{19}-\mathrm{Ga}_{3}-\mathrm{O}_{11}$ |
| 6 | $f \boldsymbol{\theta}_{\mathbf{2}}$ | 0.45 | $\mathrm{O}_{11}-\mathrm{Ga}_{3}-\mathrm{O}_{8}$ |
| 7 | $f \delta_{1}$ | 0.50 | $\mathrm{O}_{5}-\mathrm{Ga}_{1}-\mathrm{O}_{7}$ |
| 8 | $f \delta_{1}$ | 0.61 | $\mathrm{O}_{5}-\mathrm{Ga}_{1}-\mathrm{O}_{9}$ |
| 9 | $f \theta_{3}$ | 0.36 | $\mathrm{O}_{11}-\mathrm{Ga}_{1}-\mathrm{O}_{14}$ |
| 10 | $f_{\gamma}$ | 0.5 | $\mathrm{Ga}_{1}-\mathrm{O}_{8}-\mathrm{Ga}_{3}$ |
| Torsion |  |  |  |
| 11 | $f \tau$ | 0.02 | Torsion around $\mathrm{Ga}_{1}-\mathrm{O}_{7}$ axis |

Stretching-stretching interactions

| $f_{\mathrm{t}_{1} \mathbf{0}_{2}}$ | 0.08 |
| :---: | :---: |
| $f_{44}$ | 0.30 |
| $f_{1,1}$, | 0.25 |
| $f_{010}$ | 0.08 |
| $f_{01} \mathrm{O}_{2}$ | -0.03 |
| $f_{\mathrm{O}_{1} \mathrm{O}_{2}}^{\prime}$ | -0.14 |
| $f_{\mathrm{O}_{2} \mathrm{O}_{2}}$ | -0.08 |

Bending-bending interactions

| $f \theta_{1} \theta_{2}$ | 0.35 |
| :--- | ---: |
| $f \theta_{2} \theta_{2}$ | 0.05 |
| $f \theta_{2} \gamma$ | 0.19 |
| $f \theta_{2} \delta_{1}$ | -0.1 |
| $f \theta_{2} \delta_{2}$ | 0.1 |
| $f \delta_{1} \delta_{2}$ | -0.2 |
| $f \delta_{1} \gamma$ | 0.15 |
| $f \gamma \gamma$ | -0.06 |

Stretching-bending interactions

| $f_{\mathrm{t}_{2}} \delta_{\mathbf{1}}$ | 0.26 |
| :--- | ---: |
| $f_{\mathrm{o}_{\mathbf{z}}} \theta_{1}$ | -0.36 |
| $f_{\mathrm{tr}}=f_{0 r}$ | 0.35 |

constant is the second derivative of $V_{\mathrm{n}}$ and is equal to $2 \beta^{2} D_{e}$; therefore the force constant is proportional to the dissociation energy $D_{e}$. Thus, when one considers the breaking of $9-16$ and $27-10$ bonds (Fig. 6), i.e., when one performs a cleavage following the ( 001 ) plane, one will have to spend a dissociation energy equal to twice $D_{(001)}=$
$0.6 / 2 \beta^{2}$ (where 0.6 is the valence force constant for this bond); on the other hand, if such a cleavage is imagined occurring along the (100) plane the dissociation energy would be twice $D_{(100)}=1.3 / 2 \beta^{2}$. Therefore one realizes that the (001) plane is an easier cleavage plane than the (001) plane.

The potential energy distribution for each mode has been obtained. Table IV reports contributions to the potential energy originating from principal force constants larger than or equal to $15 \%$. It is difficult to give a description of vibrations from the PED reported in this table, since the vibrations seem to be heavily mixed and, moreover, the internal coordinates are unappropriate to give a chemical description of vibrations; this is due to the fact that the valence force field used here does not take into account the local symmetry. In particular, atoms are distributed into layers; each atom is in a symmetry plane and therefore it should be more interesting to consider localized internal coordinates of the type of those reported in Table II.

Table IV gives a description of normal modes in terms of the localized coordinates introduced in Table II and Fig. 3. For instance, for the frequency calculated at 760 $\mathrm{cm}^{-1}$ the displacements of atoms involve motions corresponding to localized coordinates 1 and 5 (Fig. 3). As an example, four normal coordinates are represented in Fig. 8 and were chosen to cover the whole frequency range. One corresponds to a vibration which is mainly a pure asymmetric stretching mode of the $\mathrm{Ga}_{1}\left(\mathrm{O}_{\mathrm{I}}\right)_{2}$ group of atoms. It is calculated at $628 \mathrm{~cm}^{-1}$. The mode at $360 \mathrm{~cm}^{-1}$ corresponds to oxygen displacements perpendicular to the (010) plane and can be assigned to a twisting of the $\mathrm{Ga}_{\mathrm{I}}\left(\mathrm{O}_{\mathrm{I}}\right)_{2}$ group. It is worth noting that for these two normal coordinates, the gallium displacements are nearly equal to zero. On the other hand, for the modes at 195 and $113 \mathrm{~cm}^{-1}$, the gallium atom displacements are strongly involved. The $113-\mathrm{cm}^{-1}$ mode

TABLE IV
Normal Modes of $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$

| Frequency |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Obs. |  | Cal. | Assignments of the modes |  |  |  |
| Raman | ir ${ }^{\text {a }}$ |  | Symmetry | PED ${ }^{\text {b }}$ | LX ${ }^{\text {c }}$ | Description |
| 763 |  | 760 | $\mathrm{A}_{0}$ | $24 f_{t}+20 f_{0}$ | $1+2$ |  |
|  | 760 | 731 | $\mathrm{B}_{\mathbf{u}}$ | $\begin{array}{ll}24 f_{\mathrm{t}}+20 f_{\mathrm{o}} & 1+2 \\ 17 f_{\mathrm{t}}+29 \mathrm{f}_{\mathrm{o}} & \end{array}$ |  |  |
|  | 720 | 692 | $\mathrm{B}_{u}$ | $22 f_{0}$ |  | Stretching and |
|  | 668 | 656 | $\mathrm{A}_{\text {u }}$ | $62 f_{1}+16 \delta_{1}$ |  |  |
| 657 |  | 654 | $\mathrm{A}^{\underline{s}}$ | $41 f_{1}+22 f_{0}+15 \delta_{1}$ | $2+5$ | bending |
| 651 |  | 644 | $\mathrm{B}_{g}$ | $65 f_{t}+17 \delta_{1}$ | 3 | of |
|  | 640 | 626 | $\mathrm{B}_{\text {u }}$ | $64 f_{\mathrm{t}}+16 f_{0}$ |  | $\mathrm{Ga}_{1} \mathrm{O}_{4}$ |
| 628 |  | 628 | $\mathrm{A}_{\text {g }}$ | $75 f_{t}$ | 4 |  |
|  | 525 | 526 | $\mathrm{B}_{u}$ | $91 f_{1}+20 \delta_{1}+18 \delta_{2}$ |  |  |
| 475 |  | 474 | $\mathrm{A}_{\text {o }}$ | $34 f_{\mathrm{t}}+36 f_{0}+20 \delta_{1}+16 \delta_{2}$ | $1+5$ |  |
| 475 | 455 | 468 | B | $39 f_{0}+30 \delta_{1}+17 \delta_{2}$ | 6 ) |  |
|  |  | 500 | $\mathrm{A}_{\text {u }}$ | $34 f_{0}+29 \delta_{1}+15 \delta_{2}$ |  |  |
| 415 |  | 406 | $\mathrm{A}_{g}$ | $33 f_{0}+18 \delta_{2}$ | $7+9$ | Deformation of |
|  | 375 | 374 | $\mathrm{B}_{u}$ | $338_{3}$ |  | $\mathrm{Ga}_{1}\left(\mathrm{O}_{1}\right)_{2}$ |
| 353 |  | 360 352 | $\mathrm{B}_{s}$ | $59 f_{0}+32 \delta_{1}+16 \delta_{3}$ |  | Octahedron |
|  | n.o. | 352 | $\mathrm{A}_{\text {u }}$ | $16 f_{1}+64 f_{0}+25 \delta_{3}$ | + 7 | modes |
| 346 |  | 353 | $\mathrm{A}_{0}$ | $22 f_{0}+39 \delta_{3}$ | $9+7$ |  |
| 318 |  | 308 | $\mathrm{A}_{g}$ | $20 f_{\mathrm{t}}+32 f_{\mathrm{o}}+21 \delta_{2}$ |  |  |
|  | 310 | 337 | $\mathrm{B}_{u}$ | $17 f_{1}+24 f_{0}+19 \delta_{2}$ |  |  |
|  | 290 | 300 | $\mathrm{B}_{u}$ | $16 f_{\mathrm{t}}+30 f_{0}+15 \delta_{1}$ |  |  |
|  | 250 | 216 | $\mathrm{A}_{u}$ | $38 \delta_{1}+21 \delta_{2}+20 \delta_{3}$ |  |  |
| 199 |  | 195 | $\mathrm{A}_{s}$ | $29 f_{0}+32 \delta_{1}$ | 12 ) |  |
| 169 |  | 166 | $\mathrm{A}_{g}$ | $15 f_{t}+398 \delta_{1}+27 \delta_{2}+33 \delta_{3}$ | 13 | and |
|  | 155 | 155 | $\mathrm{B}_{\text {u }}$ | $26 f_{\mathrm{f}_{\mathrm{t}}}+52 f_{\mathrm{o}}+71 \delta_{1}$ |  | translations |
| 147 |  | 152 | $\mathrm{B}_{s}$ | $33 f_{4}+21 \delta_{1}+16 \delta_{2}+23 \delta_{3}$ |  | of |
| 114 |  | 114 | Bo | $31 f_{0}+81 \delta_{1}+98 \delta_{2}$ | 10 | chains |
| 111 |  | 113 | $\mathrm{A}_{g}$ | $62 \delta_{1}+64 \delta_{2}$ | 15 ) |  |

${ }^{a}$ Infrared maxima measured by transmission; for broad and strong ir bands, the frequency of the maxima lies between TO and LO frequencies, the calculated corresponding frequency is probably rather near the TO component.
${ }^{b} f_{\mathrm{t}}=$ Tetrahedron stretching; $f_{0}=$ octahedron stretching; $\delta_{1}=\mathrm{Ga}-\mathrm{O}-\mathrm{Ga}$ bending; $\delta_{2}=\mathrm{Ga}_{\mathrm{II}}-\mathrm{O}_{2}$ bending; $\delta_{3}=$ $\mathrm{Ga}_{1}-\mathrm{O}_{2}$ bending. Only contributions originating from principal force constants greater or equal to $15 \%$ are reported. n.o. $=$ not observed.
${ }^{c}$ Numbers correspond to localized coordinates described in Table I and Fig. 3 and which are mainly involved in the LX cartesian displacements.
corresponds to a collective displacement of atoms and is assigned to a translation following the $z$ axis of tetrahedra chains. The $195-\mathrm{cm}^{-1}$ mode represents librations of chains around the $y$ axis. The full description of cartesian displacements can be obtained from the authors.

## 3. Temperature Effect on the Raman <br> Spectra

The main causes which contribute to the broadening of Raman and infrared lines are the following:
(i) anharmonicity phenomena,
(ii) thermally activated reorientational


Fig. 8. Relative cartesian displacements of atoms involved in three in-plane ( $A_{0}$ ) and one out-of-plane ( $B_{a}$ ) normal modes at high and low frequency, respectively. For high-frequency modes oxygen displacements predominate, while at low frequency gallium displacements contribute to a large extent to the normal mode.
processes which, for instance, could involve librations of tetrahedra chains,
(iii) vibrational dephasing (surrounding time fluctuations), and
(iv) creation of defects leading to spectral density, which can be large for dispersive modes.

The reorientational term can be eliminated from the broadening causes; in fact, it usually plays an important role in molecular crystals in which the reorientation of individual groups of atoms or molecules can occur upon increasing the temperature (11). This is not the case in a crystal where there exists a tridimensional framework in which the reorientation of tetrahedra chains requires much energy.
(a) Temperature dependence of the halfwidths. The broadening factor of Raman bands in the $10-$ to $1000-\mathrm{K}$ temperature range is very large. Most of the bands above $300 \mathrm{~cm}^{-1}$ broaden by a factor of 15 or 20. On the other hand, the low-frequency modes are much less sensitive to the temperature effect and they broaden only by a factor of 8 .

The difference of behavior between highand low-frequency modes is associated with the difference of the types of motions
involved in these modes. Low-frequency modes involve mainly motions of weak amplitude (Fig. 8). On the contrary, high-frequency modes involve motions of large amplitude, mainly for oxygen atoms. However, it is difficult to establish a general rule. In fact, among the low-frequency modes the $147-\mathrm{cm}^{-1}$ band does not follow this behavior; its half-width varies from 2 to $17 \mathrm{~cm}^{-1}$, although it corresponds to a $T^{\prime} y$ translation of tetrahedra chains with amplitudes very similar to those calculated for the rest of the external modes. Moreover, one can also try to correlate the broadening to the displacement direction of atoms, the $B_{g}$ mode at $651 \mathrm{~cm}^{-1}$ which is assigned to displacements perpendicular to the (010) plane being less broadened than the $360-$ $\mathrm{cm}^{-1}$ mode of $B_{g}$ symmetry. Therefore it is difficult to establish a correlation without aberrant points. Except for the $B_{g}$ modes at $147 \mathrm{~cm}^{-1}$, it seems in general that the modes involving oxygen atoms of type II are less broadened than those involving oxygen atoms of type $I$; in particular, this is the case for the two $B_{g}$ modes discussed above. In the same way, among the high-frequency modes one can verify that the bands at 763 and 628 and the two modes at $475 \mathrm{~cm}^{-1}$ involving the local coordinates $1,2,4$, and 6 , all based on the $\mathrm{Ga}_{\mathrm{I}}\left(\mathrm{O}_{1}\right)_{2}$ group of atoms located in the (010) plane, become broader than the $651-\mathrm{cm}^{-1}$ mode, involving the $\mathrm{Ga}_{1}\left(\mathrm{O}_{\mathrm{II}}\right)_{2}$ group of atoms, as if the perturbation acting on the $\mathrm{GaO}_{\mathrm{I}}$ vibrators was weaker than that acting on the $\mathrm{GaO}_{\text {II }}$ ones. These vibrators, being freer from perturbation, could oscillate with a larger amplitude.
(b) Temperature dependence of frequencies. As an example, we give in Fig. 10 the temperature dependence of frequencies for three modes for which the variations are the largest ones. For comparison we report also two external modes where the frequency variations are much smaller. In the temperature range under study, variations are comparable with those observed for other


Fig. 9. Experimental Raman band half-widths as a function of temperature. (---) Experimental curve; (-) curve calculated from anharmonic contributions (see text); values of the half-widths are reported on the left and the right parts of the curves for 10 and 1070 K , respectively.


Fig. 10. Raman frequency as a function of temperature (only five representative modes are reported).
crystals. These frequency shifts are connected on the one hand to the volume variation of the material as a function of temperature ( $\delta \omega_{\mathrm{J}}$ terms) and on the other hand to the contribution of anharmonicity ( $\Delta \omega_{\mathrm{J}}$ terms) (8). The terms are connected through the relationship

$$
\omega_{\mathrm{J}}(T)=\omega_{\mathrm{J}}(O)+\delta \omega_{\mathrm{J}}+\Delta \omega_{\mathrm{J}}
$$

where $\delta \omega_{\mathrm{J}}$ can be expressed as a function of the Grüneisen coefficient (9). Because of the lack of additional data we have not been able to estimate the $\delta \omega_{\mathrm{J}}$ term associated with the volume variation. This will be undertaken in a forthcoming study (12).
(c) Broadening of Raman bands due to anharmonicity. The half-width of bands is usually expressed by the following cubic and quartic terms:

$$
\begin{align*}
& \quad \Gamma_{\mathrm{cubic}}=A\left[\left(e^{\hbar \omega_{0} / 2 k T}-1\right)^{-1}+\frac{1}{2}\right],  \tag{1}\\
& \Gamma_{\text {quartic }}= \\
& \left.B\left[\left(e^{\hbar \omega_{0} / 3 k T}-1\right)^{-1}+\frac{1}{2}\right)^{2}+\frac{1}{12}\right], \tag{2}
\end{align*}
$$

where $A$ and $B$ are two constants.
The cubic term describes the destruction of one phonon of $\omega_{0}$ frequency into two phonons which can in a first approximation be considered to have a frequency equal to $\omega_{0} / 2$.

In the same way, the quartic term can be associated with the destruction of one phonon of $\omega_{0} / 3$ frequency ( 8 ). At high temperature, the cubic term is proportional to $T$ and the quartic term to $T^{2}(8,10)$.

We fitted our data with these equations for all the observed modes (except for the mode at $353 \mathrm{~cm}^{-1}$ which is smeared out by the $346-\mathrm{cm}^{-1}$ line and for the $475-\mathrm{cm}^{-1}$ band which is a superposition of two modes of $A_{g}$ and $B_{g}$ symmetry, respectively); the corresponding $A$ and $B$ parameters are reported in Table $V$.

The cubic term dominates for the phonon of low frequency, while for high-frequency

TABLE V
Contribution of Anharmonicity to the Raman Bands Half-width ( $\Gamma$ )

| Frequency <br> $\left(\mathrm{cm}^{-1}\right)$ | $\Gamma(1000 \mathrm{~K})$ <br> $\left(\mathrm{cm}^{-1}\right)$ | $A$ <br> $\left(\mathrm{~cm}^{-1}\right)$ | $B$ <br> $\left(\mathrm{~cm}^{-1}\right)$ | $\gamma(1000 \mathrm{~K})$ | $a$ | $b$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 763 | 31 | 7.395 | 2.006 | 0.041 | 9.6 | 2.6 |
| 657 | 31 | 3.237 | 2.204 | 0.047 | 4.9 | 3.4 |
| 651 | 21 | 1.803 | 1.497 | 0.032 | 2.8 | 2.3 |
| 628 | 30 | 4.717 | 1.604 | 0.048 | 7.5 | 2.6 |
| 415 | 18 | 1.561 | 0.439 | 0.043 | 3.8 | 1.1 |
| 346 | 29 | 1.946 | 0.554 | 0.084 | 5.6 | 1.6 |
| 318 | 26 | 3.904 | 0.193 | 0.082 | 12.3 | 0.6 |
| 199 | 7 | 0.606 | 0.011 | 0.035 | 3.1 | 0.06 |
| 169 | 7 | 0.528 | 0.008 | 0.041 | 3.1 | 0.05 |
| 147 | 14 | 0.961 | 0.017 | 0.095 | 6.5 | 0.1 |
| 114 | 5 | 0.286 | 0. | 0.044 | 2.5 | 0. |
| 111 | 0.364 | 0.007 | 0.081 | 3.3 | 0.06 |  |

Note. $A$ and $B$ are the parameters which are found from the best fit between the experimental curves of Fig. 9 and the expression of $\Gamma_{\text {cublc }}+\Gamma_{\text {quartic }}$ given in the text. $\gamma, a, b$ are the $\Gamma, A, B$ parameters reduced to the frequency of the corresponding mode.
modes, the quartic term can explain by itself the shape of the curves $\Gamma=f(T)$. This can be well understood as far as the probability for the destruction of one phonon at low frequency into three other phonons of $\omega_{0} / 3$ frequency is much lower than for the high-energy phonons (8).
(d) Creation of defects upon increasing the temperature. Most of the curves present an inflection point, and for some of them a horizontal part is even observed (that is the case for the 111-, 114-, 169-, and $199-\mathrm{cm}^{-1}$ modes). The experimental points of Fig. 9 seem to be distributed above the solid-line curve representing the anharmonicity contribution to the half-width. The experimental curves coincide with the calculated curves for temperatures above 700 K . This is mainly observed for dispersive modes (external modes) and indicates that an amount of disorder appearing above 100 K could be responsible for the breakdown of the $k=0$ selection rule and this could induce a spectral density. The existence of a horizontal part shows that the rate of disorder seems to be maximum at
temperatures as low as 400 K . The disorder could originate from the creation of defects (vacancies, occupation of interstitial sites probably by oxygen atoms); in fact, the nonobservation of any peaks in the DTA experiments seems to exclude the possibility of a phase transition of the displacive type (e.g., chain reorientation). Moreover, the large range of temperature in which this broadening is observed indicates that this phenomenon is strongly of second order and should be of the order-disorder type as already observed for some molecular crystals (II). Finally, a phase transition could give rise to a frequency discontinuity near the transition temperature and this is not actually observed.

## Conclusion

We report a complete assignment of the ir and Raman frequencies of $\beta \mathrm{Ga}_{2} \mathrm{O}_{3}$ in terms of symmetry. The spectra are more consistent with the centrosymmetric structure reported by Geller than with that proposed by Wolten.

We propose a description of the normal modes based on:
(i) the potential energy distribution given in terms of internal coordinates,
(ii) the cartesian displacements of the atoms, and
(iii) localized coordinates being chemically significant.

A complete set of force constants is given. The existence of small force constants in the [100] direction is consistent with the observation of a (001) cleavage plane.

The temperature dependence of frequencies and half-widths in the 10 to $1000-\mathrm{K}$ range is mainly due to anharmonicity. The contribution of quartic terms is important for high-frequency modes, while the cubic terms are almost sufficient to explain the broadening of low-frequency lattice modes.

A noticeable increment to the broadening of Raman bands in the $100-$ to $600-\mathrm{K}$ temperature range is interpreted in terms of a reversible low rate of disorder; defects such as the occupation of interstitial sites, or a slight shift of the stoichiometry, could be put forward.

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## References

1. (a) D. Dohy and G. Lucazeal, J. Mol. Struct., in press, XV European Congress of Molecular Spectroscopy, Norwich, 1982: (b) D. Dohy, Thèse de $3^{e}$ cycle, Paris (1982).
2. Ph. Colomban and G. Lucazeal, J. Chem. Phys. 72, 1213 (1980), and G. Lucazeau, Solid State Ionics, in press (1983).
3. S. Geller, J. Chem. Phys. 33, 676 (1960).
4. G. M. Wolten and A. B. Chase, J. Solid State Chem. 16, 377 (1976).
5. N. T. McDewiti and W. L. Baun, Spectruchim. Acta 20, 799 (1964).
6. (a) A. Revcolevschi, unpublished; (b) A. Revcolevschi and M. de la Rochère, unpublished; (c) A. Revcolevschi and M. Saurat, Rev. Int. Hautes Temp. Refract. 8, 291 (1971).
7. T. Shimanouchi, Computer programs for NCT of polyatomic molecules, 1968.
8. T. Sakurai and T. Sato, Phys. Rev. B 4(2), 583 (1971).
9. A. K. Sood, A. K. Arora, V. Umadevi, and G. Venkataraman, Pranama 16, 1 (1981).
10. I. P. Ipaiova, A. A. Maradudin, and R. F. Wallis, Phys. Rev. 155(3), 882 (1967).
11. K. Chhor, G. Lucazeau, and C. Sourisseau, J. Raman Spectrosc. 11(3), 183 (1981).
12. J. R. Gavarri, and D. Dohy, J. Solid State Chem., to be published.

[^0]:    Note. These coordinates are defined in Fig. 3 and used in the description of normal modes reported in Table IV.
    ${ }^{a}$ Acoustic modes.

